49

MATHEMATICS HSSC-II

SECTION – A (Marks 20)

Time allowed: 25 Minutes

Candidate Sign. _____

Invigilator Sign. _____

Answer Sheet No. _____

Signature should not cross this line

Version No.				
4	0	0	8	1

ROLL NUMBER						

Deleting/overwriting is not allowed. ***Do not use lead pencil.***

Fill the relevant bubble against each question according to curriculum:

	Question سوال	A	B	C	D	A	B	C	D
1.	If $f(x) = 3x - 4$, then $f^{-1}(5)$ equals:	1	3	5	11	○	○	○	○
	If $f(x) = \ln(2x)$, then $f'(3)$ equals:	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{3}{2}$	○	○	○	○
	If $\int xe^x\,dx = e^x(x-1) + C$, then $\int_0^1 x\,e^x\,dx$ equals:	0	1	$e - 1$	$1 - e$	○	○	○	○
	The general solution of $\frac{dy}{dx} = \frac{x}{y}$ is:	$y^2 = x^2 + C$	$y^2 = -x^2 + C$	$y^2 = 2x^2 + C$	$y = x + C$	○	○	○	○
	For the vector function $r(t) = \langle \ln(t-1), \sqrt{4-t} \rangle$, the domain in $\mathbb{R}$ is:	(1,4)	[1,4)	(1,4]	[1,4]	○	○	○	○
	The period of $\tan(3x)$ is:	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	2π	○	○	○	○
7.	For hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$, slopes of asymptotes are:	$\pm\frac{16}{9}$	$\pm\frac{4}{3}$	$\pm\frac{9}{16}$	$\pm\frac{3}{4}$	○	○	○	○
8.	If $f(x) = \sqrt{x+1}$ for $x \geq -1$, then the range of $f^{-1}(x)$ is:	$(-\infty, 0]$	$[0, \infty)$	$(-\infty, -1]$	$[-1, \infty)$	○	○	○	○
9.	For $f(x) = x^3 - 3x^2 + 2$, the value of $f''(2)$ gives:	local maximum	local minimum	inflection point	cannot be determined	○	○	○	○
10.	For equation $x^3 - 4x - 9 = 0$, the interval in which a root lies by sign change:	(0, 1)	(1, 2)	(2, 3)	(3, 4)	○	○	○	○
11.	Volume generated by revolving the region under $y = \sqrt{x}$ from $x = 0$ to $x = 4$ about the x-axis is:	$4\hat{x}$	6π	$8\hat{x}$	10π	○	○	○	○
12.	The area of the triangle with vertices $(0,0)$, $(4,0)$, $(0,6)$ is:	10	12	14	24	○	○	○	○
13.	Equation $3x^2 - 5xy + 2y^2 = 0$ represents:	No real lines	Two distinct lines	A single line	Two coincident lines	○	○	○	○

	Question سوال	A	B	C	D	A	B	C	D
14.	The number of solutions of $\cos x = 0.3$ in $[0,2\pi)$ is:	0	1	2	4	○	○	○	○
15.	The slope of the tangent to the circle $x^2 + y^2 = 25$ at the point $(3,4)$ is:	$-\frac{4}{3}$	$-\frac{3}{4}$	$\frac{3}{4}$	$\frac{4}{3}$	○	○	○	○
16.	The value of $\sin^{-1}\left(\sin\frac{5\pi}{6}\right)$ is:	$\frac{\pi}{6}$	$\frac{5\pi}{6}$	$-\frac{\pi}{6}$	$\frac{\pi}{3}$	○	○	○	○
17.	The position of a drone is $r(t) = \langle 5t, 4t - t^2, 2\rangle$ (meters, t in seconds). The acceleration at $t = 2$ seconds is:	$\langle 0,-2,0\rangle$	$\langle 0,2,0\rangle$	$\langle 5,-2,0\rangle$	$\langle 5,2,0\rangle$	○	○	○	○
18.	The length of the tangent from point $(10,0)$ to circle $x^2 + y^2 = 36$ is:	12	10	8	6	○	○	○	○
19.	A side of a cube is 4 cm. If side increases by 0.1 cm, approximate increase in volume is:	1.2	2.4	3.6	4.8	○	○	○	○
20.	Average value of $f(x) = x^2$ on $[0,3]$ is:	6	5	4	3	○	○	○	○

ROLL NUMBER						

MATHEMATICS HSSC–II

50

Time allowed: 2:35 Hours **Total Marks Sections B and C: 80**

SECTION – B (Marks 48)

Q. 2 Solve the following parts. **(12 x 4 = 48)**

(Use of graph paper is not necessary. Candidates can make their own grid on answer book)

(i)	If $f(x) = \frac{2x-3}{x+1}$, find $f^{-1}(x)$. Also determine the domain and range of $f^{-1}(x)$.	04	OR	The population of a city grows at a rate proportional to its current population. In 2020, the population was 50,000. In 2025, it became 60,000. (a) Find the growth constant k. (b) Predict the population in 2030.	04
(ii)	Find $\frac{dy}{dx}$ for $y = e^{2x}\ln(x)$	04	OR	The displacement–time graph of a particle is a straight line from $(0, 2)$ to $(5, 12)$, where displacement is in meters and time in seconds. (a) Find the velocity of the particle. (b) Sketch the displacement–time graph. (c) Sketch the corresponding velocity–time graph.	04
(iii)	Evaluate $\int sin^2x\, dx$	04	OR	Determine the intervals in which $f(x) = x^3 - 3x^2 - 9x + 5$ is increasing or decreasing.	04
(iv)	Solve the separable differential equation: $\frac{dy}{dx} = \frac{y+1}{x-2}$	04	OR	Check whether the line $y = 2x + 1$ intersects, touches, or misses the circle $x^2 + y^2 = 5$.	04
(v)	For the homogeneous equation $3x^2 - 5xy + 2y^2 = 0$: (a) State whether it represents a pair of lines through the origin. (b) Find the individual line equations.	04	OR	From a point on the ground, the angle of elevation to the top of a tower is 30°. After walking 40 meters closer to the tower, the angle of elevation becomes 60°. (a) Find the height of the tower. (b) Find the original distance from the first point to the base of the tower.	04
(vi)	Solve $2\cos x + 1 = 0$ for x in $[0,2\pi)$ using periodic properties.	04	OR	Given $r(t) = \langle t^2 - 1, 3t, \ln(t+1)\rangle$, find $r'(t)$ and the magnitude of $r'(2)$.	04
(vii)	For the parabola $y^2 = -12x$, find the vertex, focus, directrix, and length of latus rectum.	04	OR	Find the equation of the median drawn from vertex $A(2, 4)$to the side joining $B(6, 2)$and $C(4, -2)$.	04
(viii)	The radius of a sphere is measured as 10 cm with a possible error of 0.05 cm. Use differentials to find the approximate percentage error in the volume.	04	OR	Use the Bisection Method starting with interval $[2, 3]$ for $f(x) = x^3 - 2x - 5$. Perform two iterations and state the new interval after each.	04
(ix)	The velocity of a particle is $v(t) = 3t^2 - 12t + 9$ m/s. (a) Find displacement from $t = 0$ to $t = 5$ seconds. (b) Find total distance travelled in the same interval.	04	OR	Evaluate $\int_0^1 x\, e^x\, dx$ using integration by parts.	04
(x)	Find the domain and range of $f(x) = \sin^{-1}(2x - 1)$.	04	OR	Find the area bounded by the curve $y = x^2 + 2$, the x-axis, and the lines $x = 1$ and $x = 3$.	04
(xi)	A whispering gallery (elliptical) has a ceiling height of 8 m at center and width 20 m. Where are the foci located?	04	OR	Find the derivative of $f(x) = \frac{x^2+1}{x^2-1}$ and simplify.	04
(xii)	A car moves along a curve with position $r(t) = \langle t^2 - 2t, t^3 - 3t\rangle$. (a) Find the velocity vector at $t = 1$. (b) Find the speed at $t = 1$.	04	OR	Let $f(x) = \sqrt{x-1}$ for $x \geq 1$. Find $f^{-1}(x)$ and verify that $f(f^{-1}(x)) = x$.	04

SECTION – C (Marks 32)

Note: Solve the following Questions. **(4 x 8 = 32)**

(Use of graph paper is not necessary. Candidates can make their own grid on answer book)

Q.3	Let $f(x) = \begin{cases} ax^2 + bx + 2, & x \leq 1 \\ 3x + a, & 1 < x < 3 \\ \frac{2x^2+b}{x}, & x \geq 3 \end{cases}$ Given that $f(x)$ is continuous at $x = 1$ and at $x = 3$, find the values of a and b.	**08**	**OR**	A circle passes through $(0,0)$, $(2,0)$, and $(0,2)$. (a) Find its equation. (b) Find the tangent at $(0,2)$. (c) Find the normal at $(0,2)$.	**08**
Q.4	Given the function $f(x) = 2Sinx + Cos2x, \quad x \in [0,\pi]$ (a) Find all critical points of $f(x)$. (b) Use the second derivative test to classify each critical point as a local maximum or local minimum. (c) Determine the absolute (global) maximum and absolute (global) minimum values of $f(x)$.	**08**	**OR**	Draw the graph of $y = \sin^{-1}(x)$ for $x \in [-2\pi, 2\pi]$. Use a proper scale. Label domain, range, intercepts, and endpoints.	**08**
Q.5	For the triangle with vertices $A(0,0)$, $B(4,0)$, $C(0,6)$, verify that the three medians are concurrent. Find the point of concurrency (centroid).	**08**	**OR**	**(a)** Resolve into partial fractions: $\frac{x^2 + 3x + 4}{(x^2 + 2x + 5)(x - 1)}$ **(b)** Hence find: $\int \frac{x^2+3x+4}{(x^2+2x+5)(x-1)}\, dx$	**08**
Q.6	Find general solution of homogeneous differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$	**08**	**OR**	A particle moves such that its position vector is $r(t) = 4\cos(2t)\,\hat{\imath} + 4\sin(2t)\,\hat{\jmath}$ (a) Find the velocity and acceleration vectors. (b) Show that the acceleration is directed opposite to the position vector. (c) Find the speed of the particle and the magnitude of its acceleration.	**08**