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MATHEMATICS HSSC-II

SECTION – A (Marks 20)

Time allowed: 25 Minutes

Candidate Sign. _____

Invigilator Sign. _____

Answer Sheet No. _____

Signature should not cross this line

Version No.				
4	2	0	8	2

ROLL NUMBER						

Deleting/overwriting is not allowed. ***Do not use lead pencil.***

Fill the relevant bubble against each question according to curriculum:

Question سوال	A	B	C	D	A	B	C	D
1. Average value of $f(x)=x^2$ on $[1,3]$ is:	$\frac{28}{3}$	$\frac{26}{3}$	$\frac{13}{3}$	$\frac{14}{3}$	○	○	○	○
2. If $f(x)=4x+9$, then $f^{-1}(13)$ equals:	4	1	2	3	○	○	○	○
3. If $f(x)=\ln(4x+1)$, then $f'(2)$ equals:	9	$\frac{4}{9}$	$\frac{1}{2}$	$\frac{1}{9}$	○	○	○	○
If $\int \frac{dx}{\sqrt{4-x^2}}=\sin^{-1}\left(\frac{x}{2}\right)+C$, then $\int_0^1 \frac{dx}{\sqrt{4-x^2}}$ equals:	π	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	○	○	○	○
The general solution of $\frac{dy}{dx}=\frac{3x^2}{y}$ is:	$y^2=2x^3+c$	$y=6x^3+c$	$y^2=x^3+c$	$y^2=3x^3+c$	○	○	○	○
For the vector function $r(t)=\langle\sqrt{t+3},\ln(6-t)\rangle$, the domain in $\mathbb{R}$ is:	$[-3,6]$	$(-3,6)$	$[-3,6)$	$(-3,6]$	○	○	○	○
The period of $\sec(2x)$ is:	$\frac{\pi}{3}$	2π	$\frac{\pi}{2}$	π	○	○	○	○
For the hyperbola $\frac{y^2}{36}-\frac{x^2}{49}=1$, the slopes of the asymptotes are:	$\pm\frac{7}{6}$	$\pm\frac{36}{49}$	$\pm\frac{49}{36}$	$\pm\frac{6}{7}$	○	○	○	○
A population grows as $P(t)=1000e^{0.03}$. The population after 10 years is closest to:	2000	1000	1350	1500	○	○	○	○
10. The critical points of $f(x)=x^3-3x^2+2$ on the closed interval $[-1,3]$ are:	$x=-1,0,2$	$x=0,2$ only	$x=-1,0,2,3$	$x=0,2,3$	○	○	○	○
11. For equation $x^3+2x-10=0$, the interval in which a root lies by sign change:	(3, 4)	(0, 1)	(1, 2)	(2, 3)	○	○	○	○

	Question سوال	A	B	C	D	A	B	C	D
12.	The volume generated by revolving the region under $y = \sqrt{x}$ from $x = 0$ to $x = 16$ about the x-axis is:	256π	32π	64π	128π	○	○	○	○
13.	The area of the triangle with vertices $(0,0), (6,0), (0,9)$ is:	18	54	36	27	○	○	○	○
14.	The equation $2x^2 - 3xy - 2y^2 = 0$ represents:	No real lines	Two coincident lines	Two distinct lines	A single line	○	○	○	○
15.	The number of solutions of $\sin x = -0.2$ in $[0,2\pi)$ is:	4	0	1	2	○	○	○	○
16.	The slope of the tangent to the circle $x^2 + y^2 = 169$ at the point $(5,-12)$ is:	$-\frac{12}{5}$	$\frac{5}{12}$	$-\frac{5}{12}$	$\frac{12}{5}$	○	○	○	○
17.	The value of $\cos^{-1}\left(\cos\frac{4\pi}{3}\right)$ is:	$\frac{2\pi}{3}$	$\frac{4\pi}{3}$	$\frac{\pi}{3}$	$\frac{5\pi}{3}$	○	○	○	○
18.	The position of a particle is $r(t) = \langle 8t, 6t - 2t^2, 5\rangle$ (meters, t in seconds). The acceleration at $t = 3$ seconds is:	$\langle 0,-4,0\rangle$	$\langle 8,-4,0\rangle$	$\langle 8,0,0\rangle$	$\langle 0,4,0\rangle$	○	○	○	○
19.	The length of the tangent from the point $(0,13)$ to the circle $x^2 + y^2 = 144$ is:	$\sqrt{25}$	$\sqrt{5}$	$\sqrt{169}$	$\sqrt{313}$	○	○	○	○
20.	The radius of a sphere is 5 cm. If the radius increases by 0.2 cm, the approximate increase in volume is:	25π cm³	10π cm³	15π cm³	20π cm³	○	○	○	○

ROLL NUMBER						

MATHEMATICS HSSC–II

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Time allowed: 2:35 Hours **Total Marks Sections B and C: 80**

SECTION – B (Marks 48)

Q. 2 Solve the following parts. **(12 x 4 = 48)**

(Use of graph paper is not necessary. Candidates can make their own grid on answer book)

(i)	If $f(x)=\frac{5x-2}{x+4}$, find $f^{-1}(x)$ then determine the domain and range of $f^{-1}(x)$.	04	OR	The population of a city grows at a rate proportional to its current population. In 2020, the population was 50,000. In 2025, it became 60,000. **(a)** Find the growth constant k. **(b)** Predict the population in 2030.	04
(ii)	Find $\frac{dy}{dx}$ if $y=e^{3x}ln\left(\frac{2x}{x^2+1}\right)$	04	OR	The displacement–time graph of a particle is a straight line from $(1,-4)$ to $(6,-11)$, where displacement is in meters and time in seconds. **(a)** Find the velocity of the particle. **(b)** Sketch the displacement–time graph. **(c)** Sketch the corresponding velocity-time graph.	04
(iii)	Evaluate the integral: $\int \cos^2(2\theta)\,d\theta$	04	OR	Determine the intervals in which the function $f(x)=x^3-6x^2+9x-4$ is increasing or decreasing.	04
(iv)	Solve the separable differential equation: $\frac{dy}{dx}=\frac{y-2}{x+6}$	04	OR	Examine whether the line $y=-2x+5$ cuts, is tangent to, or lies outside the circle $x^2+y^2=17$.	04
(v)	For the homogeneous equation: $x^2-7xy+12y^2=0$ **(a)** State whether it represents a pair of lines through the origin. **(b)** Find the individual line equations.	04	OR	An engineer is designing a ramp such that it rises 6 m over a horizontal distance of 18 m. **(a)** Find the angle of inclination of the ramp with the horizontal using inverse trigonometric functions. **(b)** Find the length of the reop.	04
(vi)	Solve $2\sin x+\sqrt{3}=0$ for x in $[0,2\pi)$ using periodic properties.	04	OR	Given the vector function $r(t)=\langle\sqrt{t+4},\tan^{-1}(t),2^t\rangle$ find $r'(t)$ and the magnitude of $r'(3)$.	04
(vii)	For the parabola $x^2=-8y$, find the vertex, focus, directrix, and length of the latus rectum.	04	OR	Find the equation of the perpendicular (right) bisector of the side joining the points $M(-2,6)$and $N(4,2)$.	04
(viii)	The side length of a cube is measured as 12 cm with a possible error of 0.03 cm. Use differentials to find the approximate percentage error in the volume.	04	OR	Apply the Regula-Falsi Method to determine a real root of $x^3-5x+1=0$ in the interval $[0,1]$, correct to two decimal places.	04
(ix)	The velocity of a particle is $v(t)=4t^2-16t+12$ m/s. **(a)** Find displacement from $t=0$ to $t=3$ seconds. **(b)** Find total distance travelled in the same interval.	04	OR	Use the method of integration by parts to evaluate: $\int_1^2 x(lnx)\,dx$	04
(x)	Find the domain and range of: $f(x)=\text{cosec}^{-1}(3x+2)$	04	OR	Find area of the region bounded by the curves $y=x^2$ and $y=4x-x^2$.	04
(xi)	A satellite orbits Earth in an elliptical path with Earth's center at one focus. Major axis length = 30,000 km, minor axis length = 20,000 km. Find distance from Earth's center to ellipse center.	04	OR	Differentiate the function *w.r.t. x.* $y=\frac{2x^3-3x+1}{x^2+1}$.	04
(xii)	A particle moves along a curve with position vector: $r(t)=\langle t^3-4t,\ t^2+t\rangle$ **(a)** Find the velocity vector at $t=1$. **(b)** Find the speed at $t=1$.	04	OR	Find domain and range of the following functions: **(a)** $f(x)=e^{x-2}+3$, **(b)** $g(x)=ln(x+5)$ Write your answers in interval notation.	04

SECTION – C (Marks 32)

Note: Solve the following Questions. **(4 x 8 = 32)**

(Use of graph paper is not necessary. Candidates can make their own grid on answer book)

Q.3	Let $f(x) = \begin{cases} \frac{m}{2}x^2 - \frac{n}{3}x + 1, & x \leq 2 \\ \frac{3x+m}{4}, & 2 < x < 5 \\ \frac{x^2+2n}{x+1}, & x \geq 5 \end{cases}$ Given that $f(x)$ is continuous at $x = 2$ and at $x = 5$, find the values of m and n.	**08**	**OR**	A circle has equation: $x^2 + y^2 - 6x + 4y - 12 = 0$ **(a)** Find the equation of the tangent to the circle at the point $P(7, -1)$. **(b)** Find the equation of the normal to the circle at the same point $P(7, -1)$.	**08**
Q.4	Given the function: $f(x) = 3\sin x + \cos 2x, x \in [0, \pi]$ **(a)** Find all critical points of $f(x)$. **(b)** Use the second derivative test to classify each critical point as a local maximum or local minimum. **(c)** Determine the absolute (global) maximum and absolute (global) minimum values of $f(x)$.	**08**	**OR**	Draw the graph of $y = \cos^{-1}(x)$ for $x \in [-2\pi, 2\pi]$. Use a proper scale. Label the domain, range, intercepts, and endpoints.	**08**
Q.5	For the triangle with vertices $A(1,1), B(9,1), C(1,9)$, verify that the three right bisectors are concurrent. Find the point of concurrency (circumcenter).	**08**	**OR**	**(a)** Resolve into partial fractions: $\frac{3x^2 + 2x + 7}{(x^2 + 4x + 5)(x - 3)}$ **(b)** Hence find: $\int \frac{3x^2+2x+7}{(x^2+4x+5)(x-3)}\, dx$	**08**
Q.6	Find the general solution of the homogeneous differential equation: $\frac{dy}{dx} = \frac{x^2 + 3y^2}{xy}$	**08**	**OR**	A particle moves such that its position vector is: $r(t) = 2\cos(6t)\,\hat{i} + 2\sin(6t)\,\hat{j}$ (a) Find the velocity and acceleration vectors. (b) Show that the acceleration is directed opposite to the position vector. (c) Find the speed of the particle and the magnitude of its acceleration.	**08**